

# Monday 20 May 2019 – Afternoon

## AS Level Further Mathematics A

### **Y535/01** Additional Pure Mathematics

#### Time allowed: 1 hour 15 minutes



#### You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

#### You may use:

• a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $gm s^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

#### INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are reminded of the need for clear presentation in your answers.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 4 pages.

#### Answer **all** the questions.

- 1 In decimal (base 10) form, the number *N* is 15260.
  - (a) Express *N* in binary (base 2) form. [1]

[2]

[8]

(b) Using the binary form of *N*, show that *N* is divisible by 7.

2 (a) The convergent sequence  $\{a_n\}$  is defined by  $a_0 = 1$  and  $a_{n+1} = \sqrt{a_n} + \frac{4}{\sqrt{a_n}}$  for  $n \ge 0$ . Calculate the limit of the sequence. [1]

(b) The convergent sequence  $\{b_n\}$  is defined by  $b_0 = 1$  and  $b_{n+1} = \sqrt{b_n} + \frac{k}{\sqrt{b_n}}$  for  $n \ge 0$ , where k is a constant.

Determine the value of k for which the limit of the sequence is 9. [3]

- 3 The non-zero vectors **x** and **y** are such that  $\mathbf{x} \times \mathbf{y} = \mathbf{0}$ .
  - (a) Explain the geometrical significance of this statement. [2]
  - (b) Use your answer to part (a) to explain how the line equation  $\mathbf{r} = \mathbf{a} + t \mathbf{d}$  can be written in the form  $(\mathbf{r} \mathbf{a}) \times \mathbf{d} = \mathbf{0}$ . [2]
- 4 The sequence  $\{u_n\}$  is defined by  $u_1 = 1$  and  $u_{n+1} = 2u_n + n^2$  for  $n \ge 1$ .

Determine  $u_n$  as a function of n.

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5 The tetrahedron *T*, shown below, has vertices at O(0, 0, 0), A(1, 2, 2), B(2, 1, 2) and C(2, 2, 1).



Show that the surface area of T is  $\frac{1}{2}\sqrt{3}(1+\sqrt{51})$ . [8]

- 6 (a) Determine all values of x for which  $16x \equiv 5 \pmod{101}$ . [4]
  - (b) Solve
    - (i)  $95x \equiv 6 \pmod{101}$ , [2]
    - (ii)  $95x \equiv 5 \pmod{101}$ . [2]
- 7 You are given the set  $S = \{1, 5, 7, 11, 13, 17\}$  together with  $\times_{18}$ , the operation of multiplication modulo 18.

(a)	Complete the Cayley table for $(S, \times_{18})$ given in the Printed Answer Booklet.	[4]
(b)	Prove that $(S, \times_{18})$ is a group. (You may assume that $\times_{18}$ is associative.)	[3]
(c)	Write down the order of each element of the group.	[2]
( <b>d</b> )	Show that $(S, \times_{18})$ is a cyclic group.	[1]
(e)	(i) Give an example of a non-cyclic group of order 6.	[1]
	(ii) Give one reason why your example is structurally different to $(S, \times_{18})$ .	[1]

#### **Turn over for question 8**

Helicopter *P* moves on the line  $\mathbf{r} = \begin{pmatrix} 2+4p \\ -3+p \\ 1+3p \end{pmatrix}$  and helicopter *Q* moves on the line  $\mathbf{r} = \begin{pmatrix} 5+8q \\ 2+q \\ 5+4q \end{pmatrix}$ .

The function z denotes  $(PQ)^2$ , the square of the distance between P and Q.

- (a) Show that  $z = 26p^2 + 81q^2 90pq 58p + 90q + 50.$  [3]
- (b) Use partial differentiation to find the values of p and q for which z has a stationary point. [4]
- (c) With the aid of a diagram, explain why this stationary point must be a minimum point, rather than a maximum point or a saddle point. [2]
- (d) Hence find the shortest possible distance between the two helicopters. [2]

The model is now refined by modelling each helicopter as a sphere of radius 0.5 units.

(e) Explain how this will change your answer to part (d). [2]

#### **END OF QUESTION PAPER**



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