OCR
Oxford Cambridge and RSA

Monday 20 May 2019 – Afternoon

AS Level Further Mathematics A

Y535/01 Additional Pure Mathematics

Time allowed: 1 hour 15 minutes

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You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 In decimal (base 10) form, the number N is 15 260.

(a) Express N in binary (base 2) form. **[1]**

(b) Using the binary form of N, show that N is divisible by 7. **[2]**

2 **(a)** The convergent sequence $\{a_n\}$ is defined by $a_0 = 1$ and $a_{n+1} = \sqrt{a_n} + \dfrac{4}{\sqrt{a_n}}$ for $n \geqslant 0$.

Calculate the limit of the sequence. **[1]**

(b) The convergent sequence $\{b_n\}$ is defined by $b_0 = 1$ and $b_{n+1} = \sqrt{b_n} + \dfrac{k}{\sqrt{b_n}}$ for $n \geqslant 0$, where k is a constant.

Determine the value of k for which the limit of the sequence is 9. **[3]**

3 The non-zero vectors $\mathbf{x}$ and $\mathbf{y}$ are such that $\mathbf{x} \times \mathbf{y} = \mathbf{0}$.

(a) Explain the geometrical significance of this statement. **[2]**

(b) Use your answer to part **(a)** to explain how the line equation $\mathbf{r} = \mathbf{a} + t\,\mathbf{d}$ can be written in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$. **[2]**

4 The sequence $\{u_n\}$ is defined by $u_1 = 1$ and $u_{n+1} = 2u_n + n^2$ for $n \geqslant 1$.

Determine u_n as a function of n. **[8]**

5 The tetrahedron T, shown below, has vertices at $O(0, 0, 0)$, $A(1, 2, 2)$, $B(2, 1, 2)$ and $C(2, 2, 1)$.

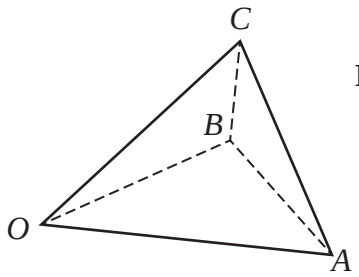

Diagram not drawn to scale

Show that the surface area of T is $\frac{1}{2}\sqrt{3}(1+\sqrt{51})$. **[8]**

6 **(a)** Determine all values of x for which $16x \equiv 5 \pmod{101}$. **[4]**

(b) Solve

(i) $95x \equiv 6 \pmod{101}$, **[2]**

(ii) $95x \equiv 5 \pmod{101}$. **[2]**

7 You are given the set $S = \{1, 5, 7, 11, 13, 17\}$ together with $\times_{18}$, the operation of multiplication modulo 18.

(a) Complete the Cayley table for $(S, \times_{18})$ given in the Printed Answer Booklet. **[4]**

(b) Prove that $(S, \times_{18})$ is a group. (You may assume that $\times_{18}$ is associative.) **[3]**

(c) Write down the order of each element of the group. **[2]**

(d) Show that $(S, \times_{18})$ is a cyclic group. **[1]**

(e) **(i)** Give an example of a non-cyclic group of order 6. **[1]**

(ii) Give one reason why your example is structurally different to $(S, \times_{18})$. **[1]**

Turn over for question 8

8 The motion of two remote controlled helicopters P and Q is modelled as two points moving along straight lines.

Helicopter P moves on the line $\mathbf{r} = \begin{pmatrix} 2 + 4p \\ -3 + p \\ 1 + 3p \end{pmatrix}$ and helicopter Q moves on the line $\mathbf{r} = \begin{pmatrix} 5 + 8q \\ 2 + q \\ 5 + 4q \end{pmatrix}$.

The function z denotes $(PQ)^2$, the square of the distance between P and Q.

(a) Show that $z = 26p^2 + 81q^2 - 90pq - 58p + 90q + 50$. **[3]**

(b) Use partial differentiation to find the values of p and q for which z has a stationary point. **[4]**

(c) With the aid of a diagram, explain why this stationary point must be a minimum point, rather than a maximum point or a saddle point. **[2]**

(d) Hence find the shortest possible distance between the two helicopters. **[2]**

The model is now refined by modelling each helicopter as a sphere of radius 0.5 units.

(e) Explain how this will change your answer to part **(d)**. **[2]**

END OF QUESTION PAPER